Beliefs Old and New

Introduction

Modern decision theory is full of talk about subjective probabilities and utilities. While these concepts fill something like the functional roles traditionally filled by belief and desire in philosophical discourse, the relationship between the old and new realizers is usually left unclear. A central piece of this puzzle is the relationship between subjective probability and belief. We shall defend a simple solution to this piece of the puzzle: X believes that \( p \) iff X’s subjective probability of \( p \) is 1. This view has been widely rejected, but there are telling responses to the standard arguments against it. Since probabilities are measures over possibility spaces, and the possibility space in question is determined by context, we may appeal to contextual features to explain away the absurd apparent consequences of our theory. Meanwhile, it is relatively easy to make a powerful case for our theory. Various functional roles are canonically associated with the concept of belief, as it figures in philosophical discussions. Belief is that psychological state which is a necessary condition for propositional knowledge. A belief that \( p \) is the psychological state that disposes linguistically competent people to assert \( p \) insofar as they desire to be sincere and believe that the audience would like to be informed whether \( p \). Belief is a state that satisfies various conceptual truths connecting belief, desire and action, such as:

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1 We are grateful to Alan Hájek, Lloyd Humberstone, Frank Jackson, Graham Oppy, Mark Scala, Ted Sider and Bob Van Gulick for helpful comments and conversation.

2 Objectivists about probability should prefer the phrase ‘degree of belief’ to ‘subjective probability’. The latter is about as apt as ‘subjective truth’ for ‘belief’. But it has by now acquired the status of an idiom, so qualms about its compositionality are perhaps pedantic.
If you want p and believe that doing X is the means to achieving p then, ceteris paribus, you will do X. Belief is a state that is governed by various rules of inference, such as: If you believe p and believe q then you ought to believe p and q. Platitudes of this sort make up what we might call ‘the canonical belief profile’. We can test an account of belief that is couched in the language of modern decision theory by seeing whether and to what extent the state so characterised fits the canonical belief profile. We show that on this score, our account fares very well. Other accounts fare less well.

In section one we set out our theory and show how it finesses some potential refutations. The next three sections display the merits of our theory.

1. Subjective Probability and Context

A famous economist, Professor Uncert, is scheduled to talk tonight at a nearby university on advances in decision theory. I want to hear the talk, but Uncert is notoriously unreliable. She often cancels papers at the last minute, and sometimes simply fails to show up. I prefer hearing the talk to staying home, but I also prefer staying home to driving to the university for no performance. To decide what to do, I must consider how likely it is the talk will take place, how much I value hearing the talk, and how much I disvalue travelling if the talk is cancelled. According to received ideals, my considerations will yield three numbers, pr, v1 and -v2, and I should go iff pr · v1 > (1 - pr) · v2. In practice, the probability and the utilities will be vague, and the decision rule may be more complicated, but the principle is right.

When I am trying to determine pr, should I consider the possibility that aliens have landed and destroyed the university, thus forcing the paper to be cancelled? Should I consider the possibility that random quantum events have seen the lecture hall transformed into a weeping statue? Obviously there is no

3 Others things will not be equal when the agent has stronger competing desires, or else is paralysed, or else is suffering from a compulsion, and so on.
requirement that I consciously consider these possibilities, and they may not even merit unconscious consideration. What it seems is reasonable, and what it seems I actually do, is assign probabilities to some possibilities, while totally ignoring others. Given that I am finite, and there are at least beth-several possible worlds, it would be tricky for the possibility space over which my subjective probability function is defined to contain all the classes of worlds there are. So we should, and do, restrict the classes of possible worlds over which our subjective probabilities are distributed.4

What is it to ignore a possibility in the sense relevant to us? We can’t just say that we believe \( p \) if we ignore the possibility that \( \neg p \), under the most natural reading of ‘ignore’, one according to which we ignore \( p \) just in case we have given no thought at all to whether or not \( p \). This will lead too quickly into assigning inconsistent beliefs to seemingly rational agents. Assume X has given no thought at all to whether or not the Bombers will win the Grand Final in 42 years time. In one good sense, X is ignoring the possibility that the

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4 Of course, if a class of worlds is contextually relevant, that does not mean that any subclass of that class is relevant: If I take seriously the proposition that there is a square thing in front of me, that renders relevant the class of worlds where there is a square thing in front of me. But I may yet completely ignore the class of worlds where there is a square dinosaur in front of me, even though the latter is a subset of the former. Put another way, I may take seriously the possibility of some proposition P without taking seriously the possibility of some proposition Q that entails P.

Lance (1995) also takes considerations about our inability to conceive of all possible worlds at once to show that Bayesian decision theory needs a non-probabilistic concept of ‘acceptance’. To make decisions using a finite machine we have to ‘accept’ that some finite set of possibilities are all the possibilities there are, even though we know full well that they are not. This argument seems a bit quick. A finite machine can consider infinitely many possibilities, provided those possibilities are finitely recursively specified. And a machine of finite size (like a human) may have either infinitely many parts or parts which are capable of being in infinitely many distinct states. Still, Lance seems to be on to something despite this gap in his argument.
Bombers win in 42 years, so by this criterion, she believes that they won’t. On the other hand she also ignores, in this sense, the possibility that the Bombers don’t win, so she believes that they will.

When we talk about ignoring a possibility, we mean to use ‘ignore’ in a more restricted sense. X ignores a possibility that \( p \) (in our sense) if (a) she ignores it in this ordinary sense and (b) she assigns probability one to a disjunction of possibilities each of which are inconsistent with \( p \). When X thinks that either Uncert will give a lecture tonight, or that Uncert will make one of her usual last minute cancellations, she implicitly rules out there being an alien invasion. She doesn’t rule out the Bombers winning in 42 years, so she doesn’t ignore that possibility, in the relevant sense.

There seems to be no need for the very same restrictions to be in place at all times. When I am asked for my opinion about the likelihood of an alien invasion, it is disrespectful to not at least consider it as a possibility. Even when it is considered, I will give it a very low probability, but that is different from totally ignoring it. The classes of worlds to which I assign positive subjective probabilities are thus determined by contextual factors.\(^5\)

To believe \( p \), we claim, is to assign subjective probability 1 to \( p \). In light of the preceding discussion, it is clear that I will do so under one of two conditions: Either I ignore the class of worlds where not-\( p \) or else I assign subjective probability zero to any contextually salient class of worlds where not-\( p \). Since one can give

\(^5\) The analysis here deliberately resembles the analysis of knowledge in Lewis (1996). On Lewis’s account, X knows that \( p \) iff X’s evidence eliminates all possible worlds in which \( \neg p \). The quantifier here is restricted to all the worlds that are not properly ignored. For the purposes of analysing the normative notion of knowledge, Lewis quite rightly insists that the relevant space of worlds has to do with not simply what is and isn’t ignored but what ought and ought not to be ignored. For the purposes of analysing the notion of belief, we should look to what is and isn’t ignored, reserving questions of propriety for the task of epistemic evaluation.
thought to a class of worlds and assign it probability zero, these ways of assigning subjective probability 1 to p need to be kept distinct.  

The theory we are advancing here is a descriptive, not a normative, claim. Hawthorne⁷ and Bovens (1999) put forward a distinct normative claim. They say that an agent X’s qualitative and quantitative states should be arranged so that there is some value q such that for all propositions p X believes p iff her subjective probability for p is at least q. Given that this is a normative claim, they presumably intend it to be a contingent fact whether such a conformity between qualitative and quantitative states does occur. (And since it is an idealisation, it will usually be false.) But it is not at all clear how they think such a claim could be false. They seem to be assuming that the qualitative and quantitative doxastic concepts refer to different parts – or states – of the mind, parts which should be in some kind of harmony, but which may swing apart. This seems ontologically extravagant; the old and new concepts of belief, we submit, provide researchers with different tools for theorising about a single subject matter. So there must be some answer to how the truth conditions for the old and new language of belief coordinate. What we offer is a particular solution to this descriptive question.

Most objections to the theory that belief is subjective probability 1 ignore the contextual element to subjective probability. The more probable we think a proposition is, the worse the odds we should be prepared to accept on it. On standard theory, if we believe p to degree 1, we should be prepared to accept a bet on p at any odds.⁸ But there are some values of p such that (a) you believe p and (b) you would not be

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⁶ It is of course a further question when it is rational to assign subjective probability zero to a salient class of worlds.

⁷ Not one of the authors of this paper!

⁸ This may not be right: at most, having a particular degree of belief is correlated with a certain disposition to accept certain bets. But the disposition may well be finkish, particularly if you think the offer of the bet is evidentially relevant. We ignore such complications. (See Lewis (1997) for an account of finkish dispositions.)
prepared to accept a bet such that you win $1 if \( p \), and get tortured if not \( p \). Hence belief is not degree of belief one\(^9\). We haven’t said much yet about what it takes for possibilities that not \( p \) to be made salient, but presumably the offer of such a bet will do it! Well, maybe not always. When we ate breakfast this morning, we ignored the possibility that there were aliens waiting to invade intending to torture all and only the breakfast eaters, and blithely gambled on eating.\(^{10}\) But normally, when a possibility that not-\( p \) is raised in the form of a more stereotypical invitation to bet, and the possibility that not-\( p \) has been previously ignored, X will revise the set of worlds over which her subjective probabilities range. Formerly her subjective probability was a measure over only \( p \)-worlds; now the measure will include not-\( p \) worlds, and they will receive non-zero probability. So she no longer believes \( p \), and hence may decline the bet.

A similar kind of objection arises from quantum indeterminacy.\(^{11}\) It seems that it is compatible with the laws of physics that a lecture hall turn into swampman, or into a statue, or into something phenomenally like an alien invasion. Indeed, it is possible to work out the probability that this will happen, given enough computing resources. Even before we do the calculations, we know that the answer will be greater than zero. So on our theory it seems that we don’t believe the lecture hall will not make one of these spectacular transformations and we don’t believe that the lecture hall will remain as a lecture hall. But this is absurd, we do believe the lecture theatre will not turn into an alien invasion, so our theory is false. Again, we insist that

\(^9\) This objection is made in Maher (1993) and Kaplan (1995).

\(^{10}\) Of course, the fact that we ate breakfast doesn’t in any sense entail that we ignored the possibility of aliens that torture breakfast eaters. We can imagine someone who attended to the possibility of such aliens, assigned it low probability, who also attended to the distinct possibility of breakfast-rewarding aliens and then decided to go ahead and eat breakfast because, inter alia, the expected disutility and utility of those respective alien possibilities cancel each other out. But this is not what we in fact do.

\(^{11}\) We are grateful to Frank Jackson here.
raising such cases changes the context, and hence the classes of worlds over which our subjective probabilities are measures. By noting these examples, we raise to salience some outlandish possibilities, and many more besides. Before they were raised no class of worlds where the lecture hall is drastically transformed received our cognitive attention. After they were raised, we assign a non-zero probability to some class of worlds of this sort. Having done this, we are merely very confident that the lecture hall will remain as a lecture hall. Soon enough, the outlandish possibilities are ignored once more and we return to doxastic slumber.

A different objection turns on the apparent inability of our theory to make the conceptually central distinction between belief and certainty. On alternative theories of belief there is at least one trivial (and initially plausible) way to make this distinction: certainty is degree of belief 1 and belief is something else. (That is what makes those theories alternatives to ours!) Every theory owes us an account of how the distinction between belief and certainty is possible, and it may seem we are the only ones in debt.\textsuperscript{12}

We suggest that certainty should be understood as robust belief. That is, X is certain that \(p\) iff X believes that \(p\), and X is not disposed to easily lose the belief that \(p\). Someone might believe \(p\), but be uncertain, because they could easily be talked out of this belief. A mere suggestion of a counter-possibility is sufficient for the set of salient possible worlds to change. Before the suggestion, all the salient worlds are \(p\)-worlds; after, some are not, and they receive positive probability. Some believers may be even more fragile: a disturbing pattern of clouds suggests a possibility that not-\(p\), and they lose belief. On our picture they count as not being certain, and this is just as intuition demands. How robust does a belief have to be to count as certain? The reader will not be surprised to learn that we favour a contextualist approach here as well.

\textsuperscript{12} This objection was suggested to us by Alan Hájek.
2. Belief, Assertion and Action

Let us consider a standard case of high probability assignment that is less than one: I own a lottery ticket and assign high probability to the proposition that it will lose. In some such case do I believe that my lottery ticket will lose? Suppose, for reductio, that we take the view that one does believe that the lottery ticket will lose. Suppose I have bought the lottery ticket for one dollar. Prior to the announcement of a winner, I am offered 10 cents for my ticket. Suppose all I care about (as far as lottery tickets go) is money. I need to choose between selling and keeping the ticket. If I believe that my ticket will lose then I believe that if I keep the ticket I will get no money. I also believe that if I sell my ticket I will get 10 cents. Now isn’t it a central part of philosophical lore that if one believes that doing X will get you no money and doing Y will get you some money and you want money then, in the absence of competing desires, one will do X (unless one loses those beliefs and desires in the interim)? Assume that I believe my ticket will lose and the appropriate folk psychological expectation will be that I sell the ticket for 10 cents. Obviously, though, no one will expect me to sell my ticket. So the assumption that I believe my lottery ticket will lose fails to square with that portion of the canonical belief profile connecting belief and action. Insofar as we thought that belief amounted to high probability assignment, we are left embarrassed. It seems that insofar as belief is to play its canonical role with respect to action, it must be something more than high probability assignment. Our account offers a decision-theoretic state that does the job.

Another fragment of the canonical profile connects belief with assertion: Belief is the psychological state that disposes one to assert \( p \) when one wishes to be sincere and believes that one’s audience would like to be informed whether \( p \). In that connection, our theory offers a very smooth explanation of some intuitive data points. Professor Reli is scheduled to give a talk on epistemology at the same time as Professor Uncert. Reli has been known to cancel talks at the last minute, but only in exceptional circumstances. We think (1)
could be sincerely asserted by someone who knows these facts, though we are somewhat less certain about (2).

(1) Professor Reli will give a talk on epistemology tonight.

(2) Professor Uncert will give a talk on decision-theory tonight.

These cases are in stark contrast to discussions about lotteries. Professor Gamble will attend neither talk because she will be home watching the lottery results, hoping that her ticket will win. The chances of this are rather remote, for in our state lotteries are conducted fairly. Still, someone has to win, and whoever it is has about the same chance as Gamble. Knowing these facts, we are not disposed to assert (3).

(3) Professor Gamble has a lottery ticket that will lose.

(3) is more probable than (1), all things considered, but we are disposed to assert (1) and not (3). (3) is much more probable than (2), but we have a greater propensity to assert (2) than (3). Notice, though, that if we make salient various possibilities in which Reli doesn't show up, then we will lose the disposition to assert (1). If it is explained that Reli fails to show up every one in twenty-five talks, then an assertion of (1) in that context will convey special insight into his circumstances on this particular occasion. If it is explained that there is a quantum mechanical probability that the matter of which Reli is made will turn into a large candlestick in the near future, then an assertion of (1) in that context will convey special insight that rules out candlestick conversion on this occasion.

Why is (3) hardly ever sincerely asserted? Why, in some special contexts, is (1) not sincerely asserted? Our account has a natural answer to these questions, namely, that in the cases where we imagine not bringing
ourselves to assert (1) and (3) respectively, we imagine ourselves lacking a belief in the relevant propositions.

Very high probability is not yet probability one: so in the ordinary case we do not yet believe that the lottery ticket will lose. In that context we merely believe it very likely that Gamble’s lottery ticket will lose.

Meanwhile, attention to such possibilities as that Reli turns into a candlestick render one such that one merely believes it very likely that Reli will show up.

Let us contrast this story with those competing accounts of belief that allow that high probability assignment short of 1 is standardly enough for belief\textsuperscript{13}. Let us call these probabilist accounts. The probabilist is faced a puzzle: One oughtn’t to able to assert that Professor Gamble will lose the lottery. However, high probability assignment to the claim that Professor Gamble will lose the lottery is epistemically beyond reproach. If such an assignment counts as belief then why not go ahead and assert that he will lose? It thus appears that our account preserves that segment of the canonical profile connecting belief with assertion while the probabilist’s does not.

One probabilist reply might be that such an assertion would be uninformative: one’s audience already knows that Gamble has a lottery ticket and one’s audience knows that lottery tickets almost never win. No one thinks that belief generates assertion when the assertion is reckoned uninformative. But suppose that one’s audience knows that Gamble has just spent some money and doesn’t know what he has spent money on. One’s audience asks whether he will be getting anything for his money. One can’t in that context assert

\textsuperscript{13} Such a view is quite prevalent in the literature. See, for example, Foley (1992) and Hall (1999). It can be traced back to Locke’s \textit{Essay Concerning Human Understanding} (Book IV, Chs. xv-xvi), though of course this requires some interpretative licence, since Locke didn’t have our modern concept of subjective probability.
"Gamble will be getting nothing back for his money", even though you and not the audience know that he has spent his money on an item that in all probability will net him nothing.\textsuperscript{14}

In the face of all this, the probabilist may make her own appeal to contextual variability: To believe $p$ is to assign suitably high probability to $p$, where what is suitable is fixed by contextual variables. In the limiting case of lottery discussions, she may say, the suitably high probability amounts to one. This response concedes that we do not, in the philosophically interesting sense, believe that our lottery ticket will lose, but offers a different diagnosis as to why this is so.

There is a systematic difficulty for such a view. The standards for the application of many predicates, such as ‘flat’ or ‘hexagonal’ are contextually variant in the way the probabilist requires. But in all these cases the contextual variability displays a feature absent in this case. We can raise the standards for proper ascriptions of ‘flat’ by drawing attention to subtle imperfections in the surfaces being studied. When we do that, our denial that the workbench is flat may be appropriate, even true. We can trivially raise the bar in this way. But for some reason lowering the bar is not so easy. Once the bar is raised we cannot trivially lower it, even if doing so would assist our conversational purposes. Consider: I can’t very well say: “The workbench isn’t flat, thanks to those subtle imperfections” and then easily go on to say “The football field is flat”.\textsuperscript{15} Suppose I have been discussing lotteries. The standards for belief in that context are very high, says our probabilist, indeed maximally high. If assertibility is context dependent in the way that the probabilist says, one should expect that having just spoken about lotteries, it would be far more difficult to assert that

\textsuperscript{14} Of course there may be SOME contexts in which one can assert some such thing. If Gamble has spent all his money on wine, women and a couple of lottery tickets, and has invested nothing, and one is asked whether Gamble’s standard of living will be improving this year, one might in that context simply ignore the possibility of his winning the lottery ticket and assert that his standard of living will, if anything, take a downturn. (Thanks to Stewart Cohen here.)

\textsuperscript{15} This point is made in Lewis (1979: 247).
Professor Reli will be giving a talk. But the conversational context of lotteries does not generate any upward pressure on the assertibility of the latter. This is exactly as one should expect given our story. It’s not that the probability threshold governing belief and assertability is shifty: It’s that certain possibilities are totally ignored and this makes for certain contents enjoying probability 1 in one’s subjective probability space and others not. One should not expect conversation about lotteries to alter the assertability of propositions about Professor Reli, since they will not, apart from exceptional circumstances\(^{16}\), affect one’s subjective probability assignments to propositions about Reli. Our account handles the data more smoothly than does the probabilist.

Let us mention some further problems for the probabilist. If it is possible to believe \(p\) while one’s degree of belief in \(p\) is less than 1, it should be permissible to sincerely assert “\(p\) and it might be that as a matter of fact not-\(p\).” But this always sounds wrong. And not only does it sound bad to say it. It seems bad to make mental judgements of that form. Given the latter datum, the badness of saying “\(p\) and it might not be that \(p\)” can’t now be explained away be appeal to conversational implicature or else to some other fragment of the theory of speech acts. Maybe the right response is to say that this can be believed, but we can never ‘say it to ourselves’, so it only seems like it can’t be believed. Or maybe this is something which can be an implicit belief, but not an explicit belief. We leave it to our opponents to choose which option seems least implausible. Our account suffers no embarrassment, meanwhile, since it holds that only someone with contradictory beliefs could be sincere in saying “\(p\) and it might be that as a matter of fact not-\(p\).’

Related points can be made about the connection between belief and knowledge. We begin with two data. First, it seems bad to either judge or assert “\(p\) and I don’t know that \(p\)”. Second, as David Hume (1739), David Lewis (1996) and others have urged, it does not at all seem that knowledge can be truly self-ascribed in

\(^{16}\) Of course, if the conversational context is like this: “Reli has a lottery ticket. He’d give up work straight away if he won the lottery,” one couldn’t then assert “Reli will be giving a talk here next week.”
a situation where there are alternatives to $p$ that are not ruled out. Insofar as one assigns some (albeit low) probability to certain alternatives to $p$, that would seem sufficient to defeat self-knowledge description. “$p$ and it might be that not-$p$” is bad. “I know that $p$ and it might be that not-$p$” is even worse. This points make trouble for the probabilist. Suppose, as the probabilist wants to allow, that we reasonably believe some $p$ while assigning it probability less than one. There will be possibilities that entail not-$p$ to which we give some credence. We will now be extremely well placed to believe “$p$ and I don’t know that $p$”. Rational high probability assignment will make for a belief that $p$ that was epistemically beyond reproach. Meanwhile, recognition of alternative possibilities that one hasn’t altogether ruled out will warrant one in believing that one doesn’t know $p$. So if belief is tantamount to high probability assignment, then beliefs of the form “$p$ and I don’t know that $p$” ought to be both commonplace and often reasonable. The world is not like that however. Since these beliefs are neither commonplace nor reasonable, an alternative account of belief is needed. Our account says that agents believe $p$ only insofar as they assign probability 1 to $p$; so construed, belief seems well suited as the psychological state which enters into knowledge and which is, indeed, governed by a constitutive knowledge norm, viz: One shouldn’t believe that which one doesn’t know.18

17 “But knowledge and probability are of such contrary and disagreeing natures, that they cannot well run insensibly into each other... ” (Hume 1739: 181)

“If you claim that S knows that $p$ and yet you grant that S cannot eliminate a certain possibility in which not-$p$, it certainly seems as if you have granted that S does not after all know that $p$. To speak of fallible knowledge, of knowledge despite uneliminated possibilities of error just sounds contradictory.” (Lewis 1996: 419).

18 Our account also, we note, note, squares happily with Timothy Williamson’s insistence that assertion is governed by a constitutive norm of knowledge, viz: Don’t assert what you don’t know. We suspect, though, that Williamson’s norm is the combined product of two more fundamental norms, viz: Don’t assert what you don’t believe and Don’t believe what you don’t know. We also note that our account squares particularly well with a contextualist account of knowledge. Just
3. Paradoxes

There are two paradoxes which need to be dealt with by any theory of belief: the lottery paradox and the preface paradox. On our theory there is a natural solution to both. We plan to show why probabilist solutions to the lottery paradox scarcely rival our own, and that our apparent difficulties with the preface can be finessed.

In many books, there is a disclaimer in the preface warning the reader about the inevitability of mistakes in what follows. Some authors are probably insincere in making this concession, some are so confident that they decline to make it, and some (notably Wittgenstein\(^{19}\)) positively insist on the correctness of everything they say. Still, those modest writers who make the concession seem not to be irrational, let alone incoherent. But since they believe for each sentence in the book that it is true, and for some sentences that they are false, they seem to have inconsistent beliefs. Whence the paradox of the preface. This paradox seems to have been first noticed, and named, by Makinson (1965).\(^{20}\)

The state lottery has millions of tickets. There is no serious chance for any of them to win. In fact, for each ticket, we may well believe that it won’t win. But we believe that some ticket will win. And this all seems perfectly rational, if apparently inconsistent. Whence the paradox of the lottery. This paradox seems to have been first noticed, and named, by Kyburg (1961).

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\(^{19}\) "... the truth of the thoughts that are here set forth seems to me unassailable and definitive." Wittgenstein (1961: 5).

\(^{20}\) As Hawthorne and Bovens (1999) note, the preface paradox has a self-referential version, where the disclaimer is taken to be part of the book. But there are problems enough with the alternative version, where the disclaimer is about the rest of the book. We have bit off enough without trying to solve paradoxes of self-reference.
The easy solution to Kyburg’s paradox is that the premises can be safely rejected. We usually don’t believe of a particular ticket, say our ticket, that it will lose. If we did believe that, we’d have nothing to say to a person who argues, “Well, if it’s going to lose, why don’t you throw it away.” We do believe that it will most likely lose, but not that it will lose.

So there is a straight solution to the paradox. Fancier solutions invite fancier counterexamples. Kyburg (1970), for example, diagnoses conjunctivitis in the lottery paradox. To generate the paradox, he notes, we require that beliefs should be closed under conjunction. This requirement, he claims, is unreasonable. We disagree. Consider: Professor Uncert didn’t present her paper, and has been court-martialled for her poor performance. She told a long-winded tale about how nature conspired to prevent her talking, and is now being cross-examined by sceptical prosecutor Rumpole.

R: From what you said, you expect us to believe that you blew a tire on the way to the talk.

U: That’s correct.

R: And you expect us to believe that vultures swooped down and ate the spare before you replaced it.

U: That’s also correct.

R: And you even expect us to believe that an electromagnetic storm prevented your cell phone from working so you couldn’t call for help.

U: Three in a row.

R: So you expect us to believe that you blew a tire on the way to the talk, vultures swooped down and ate the spare before you replaced it and an electromagnetic storm prevented your cell phone from working so you couldn’t call for help.

U: No, and I don’t think anything I said commits me to that.
Let’s hope Kyburg isn’t on jury duty!

The problem doesn’t obviously go away if we add a few more steps to the story. So if Rumpole and Uncert went through, say, six such questions before the long conjunction, Uncert’s final answer would still sound absurd. The basic inferential norms that we take to govern belief – and which are part of the canonical profile – are not easily jettisoned. Better to opt for our straight solution to the lottery paradox.

The preceding discussion raises a problem for the probabilist solution to the lottery paradox that has been offered by Frank Jackson. He holds that “the lottery paradox is a kind of Sorites paradox. ‘Heap’ is a vague term. Similarly, ‘acceptable’ is a vague term. The probability must be high, but just how high is vague and depends in a vague way on the context. Consequently no clear counterexample to ‘A is acceptable; B is acceptable; therefore, (A & B) is acceptable’ can be produced.” (1987: 134). An appeal to vagueness will be of little help here. Let us take the following as a datum: If it is clear that you believe each of six conjuncts then it is irrational to clearly fail to believe the conjunction of those conjuncts after considering it. Perhaps we don’t have any intuitions of this sort if we are considering, say, a thousand conjuncts, but it is hard to resist such intuitions when only small numbers of conjuncts are in play. The probabilist – even one institutes penumbral cases for ‘believes’ – will have a hard time respecting our datum. Suppose that the borderline between what is believed and what isn’t is vague, so S believes that p iff the probability of p is greater than x, does not believe that p iff the probability of p is less than y, where y < x, and if the probability of p is between y and x, then there is no clear fact of the matter as to whether S believes that p. If the probability of each of the original answers was greater than x, and the probability of the conjunction is less than y, it should be reasonable for you to clearly believe each of the conjuncts and yet clearly refrain from believing the conjunction when it is put to you. If y > 3x - 2\(^2\), then Uncert’s answers above should sound coherent\(^2\). Since they don’t, y is no

\(^{21}\) Or equivalently: 1 - 3 \cdot (1 - x).
greater than $3x - 2$. By lengthening the story, we can put further bounds on $y$. If an interchange similar to that above where Rumpole asks $n$ questions and then asks about the conjunction sounds odd, then $y$ must be at least $n$ times as far from 1 as $x$ is. Let us suppose that the probabilist thinks that $x$ is high but clearly less than 1: Let us suppose, in keeping with the spirit of probabilist thinking, that $x$ is high but clearly short of 1, say 0.9. Rumpole asks six questions. If our datum is to be respected, then it is irrational for Uncert to clearly fail to believe the conjunction of her answer. Suppose Uncert assigns 0.9 to each conjunct and 0.4 to the conjunction, which by everyone’s lights, is a perfectly rational assignment. For our datum to be respected, $y$ must be set below 0.4. But that is absurd. Consider someone who assigns probability 0.5 to some proposition $p$ and 0.5 to $\neg p$. Intuitively this person clearly fails to believe $p$ (and clearly fails to believe $\neg p$). But if $y$ is 0.4, then this person does not clearly fail to believe $p$. Note that this argument does not assume that if one clearly believes each conjunct, one should clearly believe the conjunction. It assumes something rather weaker (given the phenomenon of vagueness), namely that if one clearly believes each conjunct, one shouldn’t clearly not believe the conjunction upon considering it. That weaker premise still gets the probabilist into trouble. A vagueness-theoretic diagnosis of the lottery paradox thus seems untenable.

We have a neat straight solution to the lottery, but it might be thought we are in trouble with the preface paradox. Solving the latter requires noticing a contextual shift. When we are writing a book, we consider possibilities in which the facts about our subject matter differ. So a writer on the causes of the French Revolution considers different possible ways the Revolution could have come about, and then tries to

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22 In general for any values $x_1, \ldots, x_n$ there is a probability function $Pr$ such that for propositions $p_1, \ldots, p_n$, we have $Pr(p_i) = x_i$ for $i \in \{1, \ldots, n\}$ and $Pr(p_1 \& \ldots \& p_n)$ equal to the maximum of 0 and $(x_1 + \ldots + x_n) - (n - 1)$. This can be easily proved by induction on $n$. So if $y > 3x - 2$, there is a probability function such that the probability of each of Uncert’s answers is greater than $x$, and the probability of their conjunction is less than $y$. We employ the same result throughout this paragraph.
work out which combination of these is actual. When writing a preface, and particularly when writing a disclaimer, our subject matter changes. And with a change in focus comes a change in salient possibilities. Now we are talking about ourselves, and we must consider the possibility of our fallibility. When writing about Robespierre’s understanding of Rousseau, our fallibility is not a concern, though of course Robespierre’s is. That change of focus is sufficient to change the context, and when the context shifts different worlds must be ruled out for there to be belief. Thus when writing the book we can believe each sentence is true, but when considering it as our work, we can believe that some of the sentences are false.

4. Ramsey Degrees

An essential task for a subjectivist theory of probability is explaining what it means to say that X believes p to degree r. Since the subjectivist theory was developed under the shadow of positivism, the early attempts to do this were resolutely operational. While the task is essential, it is not essential that it be done with merely those resources. Ramsey made the largest advance towards getting an operational definition, in his well known (1926). In a less well known work Ramsey provides a start towards a more general definition, one suitable for those who think there is no available operational definition.

In fact the notion of a belief of degree 2/3 is useless to an outside observer, except when it is used by the thinker who says, “Well, I believe it to an extent 2/3”, i.e. (this at least is the most natural interpretation) “I have the same degree of belief in it as in p/q when I think p, q, r equally likely and know that exactly one of them is true.” (Ramsey 1929: 95)

This strongly suggests a more general principle, call it (N) for numerical.
(N) For all integers m, n, propositions p, sequences of propositions \(<q_1, \ldots, q_n>\) and agents X: X believes p to degree m/ n iff she believes it to the same degree as she would believe \(q_1 \lor \ldots \lor q_m\) were the following conditions met:

(a) X believes \(q_1 \lor \ldots \lor q_n\)

(b) For each distinct \(i, j \in \{1, \ldots, n\}\), X believes \(\neg(q_i \& q_j)\)

(c) For each \(i, j \in \{1, \ldots, n\}\), X believes \(q_i\) to the same degree as \(q_j\)

The right-hand side of (N) includes no references to numerical degrees of belief, just to comparative degrees, so this may be the start of a reduction. Whether this is true or not, (N) seems to express a platitude. To believe p to degree m/ n is just to think it is just as probable as the winning ticket in an n ticket lottery will be in the first m. Since (N) is a platitude, substitution instances of it are probably true. Setting \(m = 1\) and \(n = 1\) we get:

(N1) For all propositions p, q₁ and agents X: X believes p to degree 1 iff she believes it to the same degree as she would believe q₁ were the following conditions met:

(a) X believes q₁

(b) For each distinct \(i, j \in \{1\}\), X believes \(\neg(q_i \& q_j)\)

(c) For every \(i, j \in \{1\}\), X believes \(q_i\) to the same degree as \(q_j\)

Since (b) and (c) are trivial, we get that X believes p to degree 1 iff she believes it to the same degree as she would believe q₁ were she to believe it. If X believes p to degree x, and believes q₁ to the same degree, she believes q₁ to degree x. So for any q₁, were she to believe it she would believe it to degree 1. And that is what we aimed to prove all along.
5. Conclusion and Disclaimers

We have marshalled a variety of theoretical considerations in favour of the simple view that to believe \( p \) is to assign subjective probability 1. But doesn’t such a view collapse in favour of all too obvious linguistic counter-evidence? I am confident that the Crows will win, but not highly confident. Someone asks me if the Crows will win. I hesitate to assert flat out that the Crows will win. Instead I hedge using the term ‘belief’: “I believe that the Crows will win, but I am not at all sure that they will.” Said in the right tone, the latter kind of utterance is perfectly commonplace. Furthermore, it does not seem that all such uses can be put down to the trope of exaggeration or in some other way treated as linguistically idiomatic.

As uttered in such contexts, the English sentence “He believes the Crows will win” is true. We say he doesn’t believe the Crows will win. Isn’t this incoherent? Are we not refuted?

Yes and No. Our target was the philosophers’ notion of belief, whose sense is fixed by the kinds of functional roles that philosophers canonically use to explain what a belief is. Ask any philosopher what propositional knowledge is. They will tell you, inter alia, that knowledge requires a certain psychological state which they call ‘belief’ which gets to be knowledge by virtue of being connected to the world and/or to other beliefs and experiences in the right way. If pressed about the nature of that psychological state, they will then trot out various other fragments of what we have called the canonical belief profile. This paper was motivated by the philosophical question: ‘What is that psychological state picked out by ‘belief’ in philosophical accounts of knowledge and about which philosophers gloss using various familiar platitudes?’ Convinced that this conception of belief was no will o’ the wisp, we have attempted to give a decision-theoretic gloss on what it comes to.

It will have been natural to take us to be making a further claim, namely, that our favoured decision-theoretic state is the designatum of the English word ‘belief’ in its non-idiomatic usage. That is to assume that
the term ‘belief’ as it figures in the Anglo-American philosophical canon has the same reference as the ordinary term ‘belief’. It would be nice were that so. But it isn’t. The decision-theoretic state that we have described fits the profile that gives ‘belief’ its sense in standard philosophical discussions. But the English word ‘belief’ has a much broader usage than this, as evidenced by the fact that I will often be willing to assert ‘I believe that p’ when I am unwilling assert ‘p’. Hence ‘believes that p’ in the ordinary sense does not invariably pick out a state that disposes one to assert that p. ‘Belief’ in ordinary parlance is more of a catch-all than the ‘belief’ of the philosophers. There need be no competition here. We can even let the probabilist have her day. The probabilist does a poor job at finding a realizer for the canonical profile. Perhaps she will do rather better when it comes to the ordinary term ‘belief’, provided her theory is suitably ingenious.

Some readers may now feel altogether let down. That would be an overreaction. There is a psychological state that, if things go well, gets to be knowledge. It grounds a propensity to assert. It combines with desire in familiar ways. It is governed by well-known rules of deductive inference. Wouldn’t you like to know what that state is? Now you do.

References


