What kinds of things are natural?

Brian Weatherson, 30 November 2005

A central feature of Lewis's metaphysics is that some properties are more natural than others. The natural properties play a number of crucial roles for Lewis. Sharing natural properties makes for objective similarity. At the limit, duplicates share all their natural properties, and vice versa, things whose parts have the same natural properties and which stand in the same natural relations are duplicates. The laws are the simplest, strongest summary of reality, written in the language of the natural properties. Our thought and talk is to be interpreted so we denote things that are as natural as possible. And so on.

Despite having such a central role, Lewis is never crystal clear about which properties in reality are the natural properties. He says they are discovered by fundamental physics. But fundamental physics for centuries (if not millennia) has been in the business of discovering *magnitudes*, not *properties*. To some extent this changed in the 20th century, but not to a great degree. How are we meant to fit property talk into physics language? (This question, along with the unsatisfactory nature of the answers, is discussed in John Hawthorne's manuscript "Quantity in Lewisian Metaphysics".)

One option is to say that for every magnitude, e.g. mass, charge, momentum, there is a property of possessing that magnitude to some degree. Lewis seems to suggest this on page 64 of *Plurality* when he talks about their being a universal or trope of mass. But this can't be right, for the fact that two things both have mass is *not* a way in which they are fundamentally similar. Duplicates share their *exact* mass, not just the property of having mass.

So we need to say that the particular mass properties are the elite, natural properties. Having mass is not natural, but having a mass of 68kg is natural. This seems to be more in line with what Lewis thinks, e.g. when he talks about the mass propert*ies* in footnote 47, or talks about deriving properties by extrapolation or interpolation from instantiated properties in footnote 58. (Both footnotes are in *Plurality*.) But this can't really work either. To spell out the law of gravity in terms of the individual mass properties, we need an infinite conjunction. Or at least we might, if masses are continuous or reality is infinite. Neither of those things might be true, but they aren't the kind of things philosophers should be relying upon. (Especially since, as Lewis says in footnote 44 of *Plurality*, that naturalness is not world-relative, so the same story has to be told in an infinite world with continuum-many masses.) But that 'law' may be no simpler, at least it would be no shorter, than the full description of the universe. So it wouldn't be a law. But we shouldn't be able to prove this quickly from the armchair that the law of gravity isn't a law.

We get similar difficulties when we consider similarity. Intuitively, things that have a mass of between 67 and 68kgs are to an extent similar. Not perfectly similar, but similar. But if we take the individual mass properties to be the basic natural properties, then the property of having a mass between 67 and 68kgs is a disjunction of continuum many natural properties, and that's about as disjunctive property as one can imagine. And a property that can only be analysed as an infinite disjunction of perfectly natural properties, says Lewis, does not make for similarity amongst its instantiators.

Lewis looks to be in a bind here, and I want to suggest a moderately radical way out. The problem, as I see it, arose from taking properties to be central to our story about how things are. One way for things to be is to either have or lack a property. But another way for things to be is to have some quantity to some degree. And we shouldn't think that the second of these reduces to the first. (As we'll see, the first might reduce to the second.)

Let's take the fundamental ways things can be then to be represented not by sets, as properties are, but by functions from objects to values. A mass function is a function that takes objects as inputs, and returns numbers as outputs, with the number representing the mass of the object. There are many mass functions, one for each scale, but they all have quite a lot in common. Properties can be represented as functions as well, since we could have used characteristic functions rather than sets to represent properties.[1] They are functions into {0, 1} rather than functions into the non-negative reals, but they are still functions into values, just a rather special case.

The core idea then is that the ultimately natural and unnatural things are *functions*. Each of the mass functions is natural. The function that maps an object onto its mass if it's a fish, or it's charge if it's a fowl, is somewhat less natural. And of course there are functions that are less natural still.

Natural functions can play all the roles that natural properties played, and then some. Two things a and b are perfect duplicates iff for any natural function f, $f(a) = f(b)$. Complex things are duplicates iff their parts are duplicates and stand in the same natural relations to each other. If f is a natural function, then a and b are similar in a respect if $f(a)$ and $f(b)$ are both non-zero, and $f(a) / f(b)$ is close to one. They are perfectly similar in a respect if this ratio equals 1. A property is natural to the extent that the things in it are more like each other than the things outside it. In the limit, if there is some natural function f and some value v such that x has P iff $f(x) = $ P, then P is perfectly natural. And now we've got natural properties, we can use them for all the purposes Lewis wanted. But where natural properties won't do the trick, as in stating the laws, we can also use natural functions. So the theory becomes that laws are the simplest, strongest generalisations, where simplicity is measured in terms of the length of the statement in a language where everything denotes a natural object, property *or function*. In such a language, physical laws can be stated in one line, not as infinitary conjunctions.

I've talked so far about mass functions, but it might be wondered which function I mean. Is mass in kilograms a natural function, or mass in pounds, or mass in u or what? My preferred answer is that all of these are perfectly natural. Certainly saying that they are all perfectly natural does not affect any of the uses of those functions for the philosophical purposes described above. The scale doesn't matter to whether two things have the same value, or even to the ratio of their values. So we can effectively ignore scalar questions for these purposes. If the world is so kind as to give us a special mass (say the mass of the universe, if that's a constant, or the smallest quanta of mass, if mass are quantised) then it is tempting to say that it's a posteriori true that the scale with that special mass equal to 1 is the natural function. But since the scale doesn't matter to any of our philosophical purposes, saying this is of little or no philosophical import.

There is a seriously hard question remaining: what we should say *values* can be in the account of natural functions. So far I have only talked about numerical values, but not all physical quantities can be measured numerically. Some quantities, like momenta, take vector values. Should we say that a function from objects to vectors is a natural function?

There's a problem here, one that goes to the heart of Lewis's project of using natural properties for a range of philosophical work. The problem is that vectors relate differently to two uses of naturalness Lewis wants to make. Lewis wants to say that there are enough natural properties such that we have sufficient expressive power in a language where predicates only denote natural properties to state the laws. He also wants to say that the having or lacking of perfectly natural properties is a matter of the intrinsic nature of the object under discussion. The possibility of vector valued natural magnitudes threatens the possibility that one thing can do both jobs. It seems that a priori metaphysics shouldn't rule out the possibility that we'll need vector magnitudes to state the laws. From the viewpoint of a priori metaphysics, it is possible that something like Lorentz's Law, which uses vector cross-products in its statement and hence seems

[1] Compare Lewis. "But to me, the choice whether to take a 'way' as a unit set or its sole member seems to be of the utmost unimportance, on a par with the arbitrary choice between speaking of a set or of its characteristic function.", *Plurality*, fn 57, pg 87.

to require vector magnitudes to state, is a fundamental law.[2] But if we let vector magnitudes be intrinsic, we’ll say that one can change the intrinsic properties of a thing simply by spinning it around, and hence changing the direction its vectors are pointing.

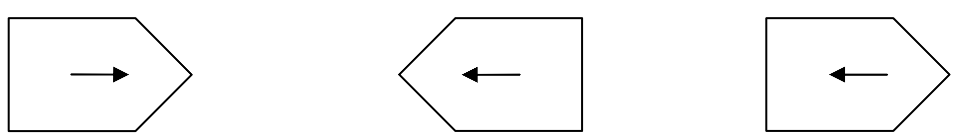
The closest Lewis gets to facing up to this dilemma is in “Humean Supervenience Debugged”, where his instinct is to grab the second horn.

> Even classical electromagnetism raises a question for Humean Supervenience as I stated it. Denis Robinson has asked: is a vector field an arrangement of local qualities. I said that qualities were intrinsic; that means they can never differ between duplicates; and I would have said offhand that two things can be duplicates even if they point in different directions. Maybe this last opinion should be reconsidered, so that vector-valued magntitudes may count as intrinsic properties. What else could they be? Any attempt to reconstrue them as relational properties seems seriously artificial. (HSD, 474)

The last line seems to involve a mistake, since as Humberstone (1996) argues, *concepts* are relational or non-relational, and *properties* are intrinsic or extrinsic. Lewis repeats something like this claim in “Zimmerman on the Spinning Sphere”.

> Let’s grant that a vector quantity associated with a spacetime point (or a point-sized bit of matter) shall count as local. Otherwise classical electromagnetism would be a problematic case for Humean supervenience, and we wouldn’t want that. (ZSS, 209).

Here he only explicitly says that vector quantities are *local* properties, and we might be able to accept that without saying they are *intrinsic* properties. Indeed, I’ll suggest below such a way we might develop that distinction, provided we can say that vector quantities are *natural* without being thereby intrinsic. We have to be careful here, however, because it does seem that vector magnitudes are somehow tied up in intrinsic properties. Imagine a world where matter is homogenous, and there is a fundamental vector valued magnitude. Consider the following three objects, where the arrow inside points in the direction the vectors associated with all the points in the objects point. (I’m just giving 2D cross sections, but imagine the objects have the same depth and the same cross sections throughout.)



A B C

Intuitively A and B are duplicates, since one can be created from the other by rotation. Neither of these is a duplicate of C, because in C’s case the (fundamental) vector magnitude points in a

[2] I’m not saying Lorentz’s Law or anything like it is fundamental in anything like this world, but the theory of natural properties and lawhood is meant to apply everywhere. It’s worth noting in this respect that the problem I’m raising for Lewis here, though it has some affinity to the problems that examples like the spinning disc raise for his Humean Supervenience, is not directed against Humean Supervenience as such. Humean Supervenience is meant to be a contingent thesis, and if fundamental vector properties are not actually realised, i.e. are alien properties in Lewis’s terminology, the difficulties they raise will not threaten Humean Supervenience. But it’s meant to be true everywhere that duplicates are things that share all perfectly natural properties, and that laws are stated in the language of natural properties. This isn’t a claim that only holds within the ‘inner sphere’.

different direction to how the object 'points'. So vector magnitudes, while not themselves intrinsic, matter for the intrinsic properties of extended objects. There isn't really a place for this kind of property in Lewis's metaphysics, as it seems there should be.

With a few modifications of the theory above, and one conceptual innovation, we can create a theory that captures all of these intuitions. I certainly don't want to claim this is the only way to capture these intuitions, but it is one approach that seems worth considering.

A natural function is a function from objects to values, where values are either numbers or vectors. Crucially, values have magnitudes. (It isn't central that there are no other kinds of function – perhaps there will be reason to expand beyond numbers and vectors. But it is crucial that the values have magnitudes.) The magnitude of a number is itself. The magnitude of a vector is its length. In either case, we'll write the magnitude of $f(x)$ as $|f(x)|$. Now we proceed as before.

Two things a and b are similar with respect to f if $|f(a)| / |f(b)|$ is close to 1. (Assuming here that both $|f(a)| / |f(b)|$ are non-zero; if either is zero they are not at all similar with respect to f.) They are perfectly similar with respect to f if that ratio equals 1. We now define some perfectly natural properties and relations, and at the end we'll define duplication.

If f is perfectly natural, then the property P such that x is P for some value v, x is P iff $|f(x)| = v$ is perfectly natural. (Equivalently we could have said that the characteristic function of P is perfectly natural.)

If G and H are intrinsic shape properties, then following function f from ordered pairs to vectors is perfectly natural. If a is G and b is H, and the centre[3] of a is not identical to the centre of b, then $f(a, b)$ is a vector from the centre of a to the centre of b, but otherwise $f(a, b) = 0$.

If f_1 and f_2 are perfectly natural vector valued functions, which are n-place and m-place functions respectively, then the following $n+m$ place function f is perfectly natural.
$f(a_1, \ldots, a_n, a_{n+1}, \ldots, a_{n+m}) = 1$ iff the direction of $f_1(a_1, \ldots, a_n)$ equals the direction of $f_2(a_{n+1}, \ldots, a_{n+m})$.

Two things a and b are duplicates iff the following two conditions are satisfied:

- For any perfectly natural one-place function f, $f(a) = f(b)$
- For any n place perfectly natural function f, if a has parts $a_1, \ldots, a_n$ such that $f(a_1, \ldots, a_n) = x$, then b has parts $b_1, \ldots, b_n$ such that $f(a_1, \ldots, a_n) = x$.

B and C above are not duplicates because, intuitively, B has a square part and a triangular part and the direction from the square part to the triangular part is the same as the direction of the fundamental vector magnitude, while C does not have parts that stand in this relationship.[4]

This explains duplication, but we also need a theory of locality in order to define Humean supervenience. This part is relatively simple though. The local properties of a region are its perfectly natural properties, and the properties it has in virtue of the perfectly natural relations amongst its parts. Two worlds are locally alike iff there is an isomorphism from small regions in one world to small regions in the other such that each region shares the same local properties as its image, and spatiotemporal relations are preserved by the isomorphism. Humean Supervenience then says that any world that is locally like this world, and is not in one or other way strange (e.g. instantiates alien properties, perhaps has different modes of persistence to the actual world) is a duplicate of this world.

The point of this exercise is not to defend all of the details of the picture just outlined. Rather, it is to show that if we allow that functions, even vector-valued functions, rather than sets are more and less natural, then we at least have the formal means to resolve some dilemmas that

[3] I'm using a fairly rough notion of 'centre' here, though I think this can be adequately formalised.
[4] That's rough, because C does have overlapping parts that stand in just that relationship. The vector from the centre of one such part to another will be shorter than the vector from the large square part of B to the large triangular part of B, so there will be a perfectly natural function that distinguishes those parts, as required.

appear irresolvable without serious cost inside Lewis's theory of natural properties. I think this is a strong reason for taking functions rather than sets to be what gets to be natural.

Acknowledgements: Thanks to John Hawthorne for letting me see his manuscript on quantities. John goes into much more detail than I do in order to show how pressing a problem this is for Lewis. Thanks also to Wolfgang Schwartz for a number of suggestions that have been incorporated into the note. Daniel Nolan has mentioned in conversation that the view being defended here was first suggested by Jack Smart. I haven't been able to trace down the references to determine whether that's true, but I do hope it is—the only thing I like more than defending Lewisian theories is defending Smartian ones!